USN | | | | | | | | | | **10MAT41**

Fourth Semester B.E. Degree Examination, June 2012

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using the Taylor's method, find the third order approximate solution at x = 0.4 of the problem $\frac{dy}{dx} = x^2y + 1$, with y(0) = 0. Consider terms upto fourth degree. **(06 Marks)**

b. Solve the differential equation $\frac{dy}{dx} = -xy^2$ under the initial condition y(0) = 2, by using the modified Euler's method, at the points x = 0.1 and x = 0.2. Take the step size h = 0.1 and carry out two modifications at each step. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$; y(0) = 1, y(0.1) = 1.1169, y(0.2) = 1.2773, y(0.3) = 1.5049, find y(0.4) correct to three decimal places, using the Milne's predictor-corrector method. Apply the corrector formula twice. **(07 Marks)**

2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at x = 0.2.

$$\frac{dy}{dx} = x + yz; \quad \frac{dz}{dx} = y + zx; \quad y(0) = 1, \quad z(0) = -1.$$ **(06 Marks)**

b. Using the Runge-Kutta method, solve the following differential equation at x = 0.1 under the given condition:

$$\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right), \qquad y(0) = 1, \qquad y'(0) = 0.5.$$

Take step length h = 0.1. **(07 Marks)**

c. Using the Milne's method, obtain an approximate solution at the point x = 0.4 of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, y(0) = 1, y′(0) = 0.1. Given y(0.1) = 1.03995, y′(0.1) = 0.6955, y(0.2) = 1.138036, y′(0.2) = 1.258, y(0.3) = 1.29865, y′(0.3) = 1.873. **(07 Marks)**

3 a. Derive Cauchy-Riemann equations in polar form. **(06 Marks)**

b. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(07 Marks)**

c. If w = φ + iy represents the complex potential for an electric field and $y = x^2 - y^2 + \frac{x}{x^2 + y^2}$ determine the function φ. Also find the complex potential as a function of z. **(07 Marks)**

4 a. Discuss the transformation of $w = z + \frac{k^2}{z}$. **(06 Marks)**

b. Find the bilinear transformation that transforms the points $z_1 = i$, $z_2 = 1$, $z_3 = -1$ on to the points $w_1 = 1$, $w_2 = 0$, $w_3 = \infty$ respectively. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where c is the circle $|z| = 3$, using Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. Obtain the solution of $x^2y'' + xy' + (x^2 - n^2)y = 0$ in terms of $J_n(x)$ and $J_{-n}(x)$. **(06 Marks)**

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(07 Marks)**

c. Prove that $\int_{-1}^{+1} P_m(x) \cdot P_n(x) dx = \frac{2}{2n+1}$, m = n. **(07 Marks)**

6 a. From five positive and seven negative numbers, five numbers are chosen at random and multiplied. What is the probability that the product is a (i) negative number and (ii) positive number? **(06 Marks)**

b. If A and B are two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, find P(A/B), P(B/A), $P(\overline{A}/\overline{B})$, $P(\overline{B}/\overline{A})$ and $P(A/\overline{B})$. **(07 Marks)**

c. In a certain college, 4% of boy students and 1% of girl students are taller than 1.8 m. Furthermore, 60% of the students are girls. If a student is selected at random and is found taller than 1.8 m, what is the probability that the student is a girl? **(07 Marks)**

7 a. A random variable x has the density function $P(x) = \begin{cases} Kx^2, & 0 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$. Evaluate K, and find: i) $P(x \le 1)$, (ii) $P(1 \le x \le 2)$, (iii) $P(x \le 2)$, iv) $P(x > 1)$, (v) $P(x > 2)$. **(06 Marks)**

b. Obtain the mean and standard deviation of binomial distribution. **(07 Marks)**

c. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. It is given that $P(0 < z < 1.2263) = 0.39$ and $P(0 < z < 1.4757) = 0.43$. **(07 Marks)**

8 a. A random sample of 400 items chosen from an infinite population is found to have a mean of 82 and a standard deviation of 18. Find the 95% confidence limits for the mean of the population from which the sample is drawn. **(06 Marks)**

b. In the past, a machine has produced washers having a thickness of 0.50 mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is found as 0.53 mm with standard deviation 0.03 mm. Test the hypothesis that the machine is in proper working order, using a level of significance of (i) 0.05 and (ii) 0.01. **(07 Marks)**

c. Genetic theory states that children having one parent of blood type M and the other of blood type N will always be one of the three types M, MN, N and that the proportions of these types will on an average be 1 : 2 : 1. A report states that out of 300 children having one M parent and one N parent, 30% were found to be of type M, 45% of type MN and the remainder of type N. Test the theory by χ^2 (Chi square) test. **(07 Marks)**

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